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ABSTRACT

In this paper the Mattson-Solomon algorithm is generalized in several directions. Based on the generalized algorithm, several classes of Bose-Chandhuri-Hocquenghem codes are given and shown to possess minimum distance of values greater than those given by the Bose-Chaudhuri-Hocquenghem found.

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I. PRELIMINARY REMARKS

It is well-known that error-correcting codes are very useful for improving the reliability of data-communication and data-storage systems. The signals transmitted in systems with coding are structured to possess certain mathematical properties. At the receiving end these properties are used to recover information from the received message by correcting erroneous symbols.

Among the many known classes of error correcting codes the most important one is a class of codes known as BCH (for Bose-Chandhuri-Hocquenghem) codes. The BCH codes form a subclass of the class of codes known as cyclic codes which can be described naturally with the concepts of polynomial algebra [Peterson 1961].

Consider the ring R of polynomials over a finite field GF(q). We may construct a residue class ring A modulo the ideal generated by the polynomial x^n-1. We shall assume in this paper that $(n,p) = 1$ where p is the characteristic of GF(q). It can be easily shown that A is an algebra of dimension n over GF(q). We shall speak of these residue classes as polynomials of degree $< n$ over GF(q). It can also be shown easily that the ideals in A are ideals generated by factors of x^n-1. These ideals are called cyclic codes. If $x^n-1 = g(x)f(x)$ then $g(x)$ is called the generator polynomial of the code and $f(x)$ its recursion polynomial. Let $v(x)$ be a code word then $v(x) = (a_o x^{n-1} + a_1 x^{n-2} + \cdots a_{n-1})$ and $v(x)$ is divisible by $g(x)$. Furthermore if $f(x) = x^k + b_1 x^{k-1} + \cdots + b_k$ then

$$a_{k+1} + b_1 a_{k+i-1} + \cdots + b_k a_i = 0 \quad i = 0,1,2 \cdots . \qquad (1)$$

It can be easily verified that x^n-1 does not have repeated factors as any such factor would not divide x^n-1 with $(n,p) = 1$. [Elspas 1959]. The solution of (1) can now be written as

$$a_j = c_1\beta_1{}^j + c_2\beta_2{}^j + \cdots + c_k\beta_k{}^j \qquad j = 0,1,2,\cdots \qquad (2)$$

with $c_i \epsilon K$ an extension field of GF(q) and $\beta_1, \beta_2, \cdots \beta_k$ roots of f(x). Given a set of values $a_o, a_1, \cdots a_k$ the values of $c_1, c_2, \cdots ,c_k$ are unique. Since the roots of f(x) are n^{th} roots of unity we may express them as powers of a primitive n^{th} root of unity β and (2) becomes

$$a_j = c_1\beta^{e_1 j} + c_2\beta^{e_2 j} + \cdots c_k\beta^{e_k j} .$$

The Mattson-Solomon polynomial $g_a(x)$ can now be defined as [Mattson and Solomon 1961]

$$g_a(x) = c_1x^{e_1} + c_2x^{e_2} + \cdots + c_kx^{e_k} .$$

It is seen that $a_j = g_a(\beta^j) \quad j = 0,1,2, \cdots$.

The Hamming weight of a polynomial is defined as the number of non-zero terms in it. A critical quantity with regard to the error-correcting capability of a code is its minimum weight, defined as the minimum of the Hamming weights of the polynomials in the code, an ideal in A. With the aid of $g_a(x)$ it is seen that the weight of v(x) is equal to n minus the number of roots of $g_a(x)$ which are also n^{th} roots of unity. Thus the problem of evaluating the weights of the code polynomial has been transformed by Mattson and Solomon to the algebraic problem of evaluating the number of roots of $g_a(x)$ which are also n^{th} roots of unity.

Let us now consider a BCH code generated by the polynomial $g(x)$ such that $g(\beta^i) = 0$, $i = m_o, m_o+1, \ldots, m_o + d_o-2$ where β is a primitive root of K; m_o and d_o are positive integers. Let us further arrange the roots of

$$g_a(s) = c_1 x^{e_1} + \ldots + e_k x^{e_k}$$

such that $0 \leq e_1 < e_2 < e_3 \ldots < e_k \leq n-1$. Since $m_o, m_o+1, \ldots,$ $m_o + d_o-2$ are not roots of $f(x)$ we may write

$$e_{m-1} < m_o < m_o + d_o-2 < e_m \leq n$$

and

$$a_i = g_a(\beta^i) = c_1\beta^{ie_1} + \ldots + c_m\beta^{ie_m} + \ldots + c_k\beta^{ie_k}$$

$$= \beta^{ie_m}[c_m + c_{m+1}\beta^{i(e_{m-1} - e_m)} + \ldots + c_{m-1}\beta^{i(e_1 - e_m)}].$$

We define $p_a(x) = c_1 x^{e'_1} + c_2 x^{e'_2} \ldots + c_k x^{e'_k}$ where $e'_i = e_i - e_m \bmod n$. The power of the leading term in $p_a(x)$ is

$$e_{m-1} - e_m = n - (e_m - e_{m-1}) \leq n - (d_o) \leq n - d_o$$

and $a_i = 0$ only if $p_a(\beta^i) = 0$. It then follows that the weight of a is

$$\begin{aligned} wt(a) &\geq n - \text{no. of zeros of } p_a(x) \\ &\geq n - \deg[p_a(x)] \\ &\geq n - (n - d_o) \\ &= d_o \ . \end{aligned}$$

We have just proved that the minimum weight of a BCH code is at least d_o.*

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*This fact had been independently proved by H. F. Mattson.

II. GENERAL RESULTS

We shall use h as the degree of extension of K over $F = GF(q)$ where K is the smallest field containing F and all the roots of x^n-1.

Let x^n-1 be factored into irreducible factors over $GF(q)[x]$. Let $f_1(x), f_2(x), \cdots, f_r(x)$ be all the irreducible factors with degree h. Let $b_1, b_2, \cdots, b_r$ be the coefficients of the (h-1)-degree term of $f_1(x), f_2(x), \cdots, f_r(x)$ respectively. We shall concern ourselves with codes whose recursion polynomials f(x) are given in the form of

$$f(x) = (x-1)\varphi_1(x)\varphi_2(x)\cdots\varphi_\delta(x)$$

where each factor of f(x) is in irreducible form. Suppose $\varphi_\delta(x)$ has degree h and $\varphi_\xi(x)$ has degree $h_\xi < h$ for $\xi = 1, 2, \cdots, \delta-1$. Furthermore we shall assume the roots of $\varphi_\delta(x)$ to be primitive n^{th} roots of unity.

We define

$$\begin{aligned} g_a(x) &= c_o + c_{11}x^{e_{11}} + c_{12}x^{e_{12}} + \cdots + c_{1h_1}x^{e_{1h_1}} \\ &\quad + c_{21}x^{e_{21}} + \cdots + c_{2h_2}x^{e_{2h_2}} \\ &\quad + \cdots \\ &\quad + c_{\delta 1}x^{e_{\delta 1}} + \cdots + c_{\delta h}x^{e_{\delta h}} \\ &= c_o + K_1(x) + \cdots + K_{\delta-1}(x) + K_\delta(x) \end{aligned}$$

and

$$p_a(x) = c_o x^{e_o'} + c_{11}x^{e_{11}'} + \cdots + c_{1h_1}x^{e_{1h_1}'}$$

$$+ c_{21}x^{e_{21}'} + \cdots + c_{2h_2}x^{e_{2h_2}'}$$

$$+ \cdots$$

$$+ c_{\delta 1}x^{e_{\delta 1}'} + \cdots + c_{\delta h}x^{e_{\delta h}'}$$

$$= c_o x^{e_o'} + k_1'(x) + \cdots + k_{\delta-1}'(x) + k_\delta'(x) \;.$$

It follows that $(n,e_{\delta k}) = 1$, $1 \leq k \leq h$.

Let us now consider the codes whose recursion polynomial satisfies one of the following two conditions:

(1) $c_{\delta k}$ is the leading coefficient of $p_a(x)$ and $c_{\delta k}^{q^{i_o}}$ is the constant term. Also $(i_o,h) = 1$, and $(n,q^{i_o}-1) = 1$.

(2) c_o is the leading coefficient of $p_a(x)$ and $c_{\delta k}$ is the constant term, or vice versa. $(n,q-1) = 1$.

Let $B_j = c_{\delta 1}\beta^{je_{\delta 1}} + \cdots + c_{\delta h}\beta^{je_{\delta h}}$. Then we have the following theorem.

<u>Theorem 1</u>: Let the code be as defined. Let conditions (1) or (2) be satisfied. If the code has minimum distance equal to d_o, then there exists a field element q_o in a field containing GF(q) such that q_oB_j will take on values $b_1,b_2,\cdots,b_r$ for a set of integers $0 \leq j \leq n-1$.

<u>Proof</u>: Case (1). Condition (1) is met.

Since the minimum distance of the code is d_o, then all the roots of $p_a(x)$ are in U, the set of all n^{th} roots of unity. Thus $c_{\delta k}^{q^{i_o}-1}$ is a product of n^{th} roots of unity, as the set of all n^{th} roots of unity forms a group.

$$(c_{\delta k}^{q^{i_o}-1})^n = 1 \quad \text{or} \quad (c_{\delta k}^{n})^{q^{i_o}-1} = 1$$

By the Reed lemma [Mattson and Solomon 1961], we can write B_j in the form

$$B_j = c_{\delta k}\beta^{je_{\delta k}} + (c_{\delta k}\beta^{je_{\delta k}})^q + \ldots + (c_{\delta k}\beta^{je_{\delta k}})^{q^{h-1}} .$$

Since $c_{\delta k}$ is the leading coefficient, $c_{\delta k} \neq 0$. This implies B_j is not identically zero. $c_{\delta k}^{\,n}$ generates a subgroup G of order R which divides $q^{i_o}-1$. Hence $c_{\delta k}^{\,n} \in GF(q^{i_o})$. As $(q^{i_o}-1,n) = 1$, $(R,n) = 1$ every element in G therefore has a unique n^{th} root. Let $q_1 c_{\delta k}^{\,n} = 1$, q_1 is in G and there exists q_o in G such that $q_o^{\,n} = q_1$. Thus we have $(q_o c_{\delta k})^n = 1$, and so $q_o c_{\delta k}$ is a n^{th} root of unity. Hence $q_o c_{\delta k}\beta^{je_{\delta k}}$ is a n^{th} root of unity for any j. Now

$$q_o B = q_o c_{\delta k}\beta^{je_{\delta k}} + q_o(c_{\delta k}\beta^{je_{\delta k}})^q + \ldots + q_o(c_{\delta k}\beta^{je_{\delta k}})^{q^{h-1}} , \quad (3.1)$$

and $c_{\delta k}^{\,n} \in GF(q^{i_o})$ and $GF(q^h)$. But $(i_o,h) = 1$. Therefore $c_{\delta k}^{\,n} \in GF(q)$ and so does q_o. Then

$$q_o\beta = q_o c_{\delta k}\beta^{je_{\delta k}} + (q_o c_{\delta k}\beta^{je_{\delta k}})^q + \ldots + q_o(c_{\delta k}\beta^{je_{\delta k}})^{q^{h-1}} .$$

$\beta^{je_{\delta k}}$ generates all the n^{th} roots of unity as j takes on all values from 0 to n-1. This implies $q_o c_{\delta k}\beta^{je_{\delta k}}$ will generate all n^{th} roots of unity when j takes all values from 0 to n-1. Since $(i_o,h) = 1$ a subset of these values of j will therefore make the expression $q_o B_j$ equal to the values of $b_1, b_2, \ldots, b_r$ when $q_o c_{\delta k}\beta^{je_{\delta k}}$ becomes a root of $f_i(x)$, $1 \leq i \leq r$. There are $r \cdot h$ values of $0 \leq j \leq n-1$ that would achieve this result.

Case (2). Condition (2) is satisfied.

The leading coefficient of $p_a(x)$ being c_o and constant term being $c_{\delta k}$ or vice versa imply that $\left(\frac{c_{\delta k}}{c_o}\right)^n = 1$, or $c_{\delta k}{}^n = c_o{}^n$. Hence $c_{\delta k}{}^n$ is in GF(q). We can by condition (2) find elements q_o and q_1 in GF(q) such that $q_1 = q_o{}^n$ and $q_1 c_{\delta k}{}^n = (q_o c_{\delta k})^n = 1$. Then

$$q_o B = q_o c_{\delta k}\beta^{je_{\delta k}} + q_o(c_{\delta k}\beta^{je_{\delta k}})^q + \ldots + q_o(c_{\delta k}\beta^{je_{\delta k}})^{q^{h-1}}$$

$$= q_o c_{\delta k}\beta^{je_{\delta k}} + (q_o c_{\delta k}\beta^{je_{\delta k}})^q + \ldots + (q_o c_{\delta k}\beta^{je_{\delta k}})^{q^{h-1}} .$$

The remaining reasoning is the same as in case (1). Q.E.D.

Let $J = \{j_{11}, j_{12}, \ldots, j_{1h}; j_{21}, j_{22}, \ldots, j_{2h}; \ldots; j_{r1}, \ldots, j_{rh}\}$ be the subset of $\{j \mid 0 \leq j \leq n-1\}$ such that $q_o B$ will take on values of $b_1, b_2, \ldots, b_r$, where q_o is as defined in the proof. Let P be the set of integers with values $b_1, b_2, \ldots, b_r$ each repeating h times. Thus $P = \{b_{11}, b_{12}, \ldots, b_{1h}; b_{21}, \ldots, b_{2h}; \ldots; b_{r1}, \ldots, b_{rh}\}$ with $b_{11} = b_{12} = \ldots = b_{1h}$, $b_{21} = b_{22} = \ldots = b_{2h}, \ldots, b_{r1} = b_{r2} = \ldots = b_{rh}$.

With the above hypothesis we know that $q_o B_j$ will take on all the values in P when j takes all values of J. Thus each of $b_{is} \in P$ is associated with exactly one value $j \in J$ which we assume already so arranged that $j = j_{is}$. By association we mean that if $f_i(\beta^{j_{is} e_{\delta k}}) = 0$ and $B^{j_{is} e_{\delta k}} + (\beta^{j_{is} e_{\delta k}})^q + \cdots + (\beta^{j_{is} e_{\delta k}})^{q^{h-1}} = b_{is}$, then b_{is} is associated with j_{is}. Let us now define partitions on the set P as follows.

<u>Definition 1</u>: A partition T_1 divides P into equivalent classes such that $b_{i_1 s_1}$ and $b_{i_2 s_2}$ are in the same class if $j_{i_1 s_1} e_{\xi 1} = j_{i_2 s_2} e_{\xi 1} \bmod n$ for $1 \leq \xi \leq \delta-1$, and $f_{i_1}(\beta^{j_{i_1 s_1} e_{\delta k}}) = 0$, $f_{i_2}(\beta^{j_{i_2 s_2} e_{\delta k}}) = 0$, $1 \leq i_1, i_2 \leq r$ where $b_{i_1 s_1}$ and $b_{i_2 s_2}$ are coefficients of the (h-1)-degree term of $f_{i_1}(x)$ and $f_{i_2}(x)$ respectively.

<u>Theorem 2</u>: In the partitioning as defined above $b_{i_1 s_1}$ and $b_{i_2 s_2}$ will be in the same equivalence class if $K_{\xi j'_{i_1 s_1}} = K_{\xi j'_{i_2 s_2}}$ for $1 \leq \xi \leq \delta-1$ and $q_o B_j(j'_{i_1 s_1}) = b_{i_1 s_1}$ $q_o B_j(j'_{i_2 s_2}) = b_{i_2 s_2}$, i.e. $f_{i_1}(q_o e_{\delta k} \beta^{j'_{i_1 s_1} e_{\delta k}}) = 0$ and $f_{i_2}(q_1 c_{\delta k} \beta^{j'_{i_2 s_2} e_{\delta k}}) = 0$ $1 \leq i_1, i_2 \leq r$.

Proof:

$$(j_{i_1s_1}e_{\xi 1} = j_{i_2s_2}e_{\xi 1}) \underset{\text{mod } n}{} \Rightarrow K_{\xi j_{i_1s_1}} = K_{\xi j_{i_2s_2}} \Rightarrow \beta^{j_{i_1s_1}e_{\xi 1}} = \beta^{j_{i_2s_2}e_{\xi 1}} \quad \text{by definition}$$

$$\Rightarrow \beta^{(j_o'+j_{i_1s_1}')e_{\xi 1}} = \beta^{(j_o'+j_{i_2s_2}')e_{\xi 1}}$$

$$\Rightarrow \beta^{j_o'e_{\xi 1}}\,\beta^{j_{i_1s_1}'e_{\xi 1}} = \beta^{j_o'e_{\xi 1}}\,\beta^{j_{i_2s_2}'e_{\xi 1}}$$

$$\Rightarrow \beta^{j_{i_1s_1}'e_{\xi 1}} = \beta^{j_{i_2s_2}'e_{\xi 1}}$$

$$\Rightarrow K_{\xi j_{i_1s_1}'} = K_{\xi j_{i_2s_2}'} \qquad \text{Q.E.D.}$$

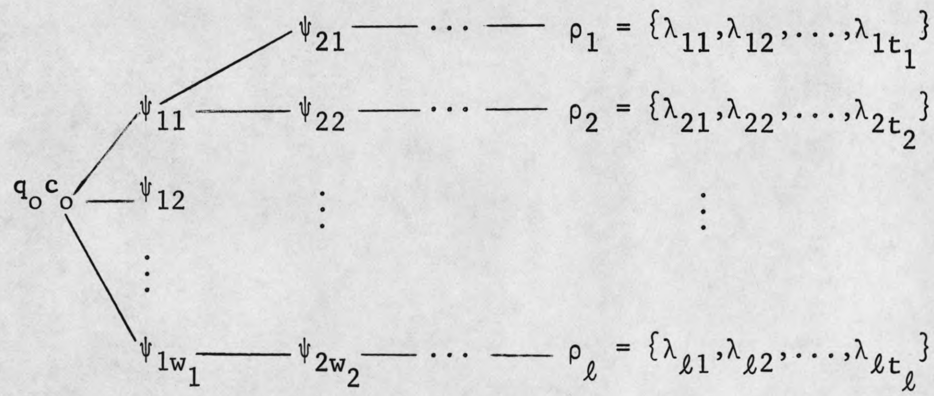
Under the partitioning process of T_1 we have P separated into equivalent classes $\rho_1, \rho_2, \ldots, \rho_\ell$ where ℓ is an integer $\leq r \cdot h$. Let t_k, $1 \leq k \leq \ell$, be the number of elements in each class ρ_k that take on the same value most frequently. Let $\sigma = \sum_{i=1}^{\ell} t_k - v_k$. We are now ready to state the main theorem.

Theorem 3: Let a code be as defined above. Suppose either condition (1) or (2) is satisfied. If $d_o < \sigma$, then the code has minimum distance $\geq d_o+1$.

Proof: The truth of this theorem is obvious if we use a graphical illustration in the form of a tree. We assume the code has minimum distance $= d_o$. Then Theorem 1 and partition T_1 are applicable. Let us consider the number of zero elements of $q_o a_j$ for $0 \leq j \leq n-1$.

$$q_o a_j = q_o c_o + K_{1j} + K_{2j} + \cdots + K_{(\delta-1)j} + q_o B_j \qquad (3.3)$$

Let us concern ourselves with only the values of $j \in J$. Let $K_{\xi j}$, $1 \leq \xi \leq \delta-1$, take w_ξ distinct values as j takes all the values in J, and let these values be $\psi_{\xi 1}, \psi_{\xi 2}, \ldots, \psi_{\xi w_\xi}$. Then we have a graph of the following form.



Of course, the elements $\lambda_{11}, \lambda_{12}, \ldots$ are just elements in the set P.

From this graph it is apparent that each class ρ_k, $1 \leq k \leq \ell$, will contribute at least $t_k - v_k$ nonzero components of $q_o a_j$, $0 \leq j \leq n-1$, when the graph is used in conjunction to Eq. (3.3). Therefore the total number of nonzero components of $q_o a_j$ when j takes all values in J is at least as large as σ. Since $q_o a_j \neq 0$ implies $a_j \neq 0$, there are at least σ nonzero a_j components. But $d_o < \sigma$. This results a contradiction. Therefore the code has minimum distance $\geq d_o + 1$.

Q.E.D.

Note that it is not necessary to show the case that the number of equivalent classes may be smaller than those we have obtained e.g. $K_{\xi 1} = 0$ for some $1 \leq \xi \leq \delta-1$. In this case we actually have a merge of equivalent classes and the contribution of nonzero components to $q_o a_j$ for $j \in J$ can only increase but never decreases.

III. SPECIAL CASES

Case (1): $f(x) = (x-1)(x-\theta)f_{\nu}(x)$, with $\theta \neq 1$,

$\theta \in GF(q)$ and $1 \leq \nu \leq r$.

Let $\theta = \beta^{e_{11}}$. Then we have

$$qe_{11} \equiv e_{11} \quad (n)$$

or

$$(q-1)e_{11} \equiv 0 \quad (n).$$

Partition T_1 in this case actually partitions the set $P' = \{b_1 b_2, \ldots, b_r\}$ into equivalent classes. We can show this by illustrating that b_i, $1 \leq i \leq r$ will always be in the same class regardless of which value of $j \in J$ is associated with b_i.

Let $q_o c_{2k} = \beta^{j_o' e_{2k}}$ and let $\beta^{(j_o'+j_i)e_{2k}}$ be a root of $f_i(x)$, $1 \leq i \leq r$. Then $(\beta^{(j_o'+j_i)e_{2k}})^q$ is also a root of $f_i(x)$ and

$$\beta^{(qj_o'+qj_i)e_{2k}} = \beta^{(j_o'+(q-1)j_o'+qj_i)e_{2k}}$$

$$= q_o c_{2k} \beta^{((q-1)j_o'+qj_i)e_{21}}$$

which states that the integer j corresponding to this second root is $(q-1)j_o' + qj_i = j_i'$. But

$$\beta^{((q-1)j_o'+qj_i)e_{11}} = \beta^{(q-1)j_o' e_{11} + qj_i e_{11}} = \beta^{qj_i e_{11}}$$

since $(q-1)e_{11} \equiv 0 \pmod{n}$. But $\theta = \beta^{e_{11}}$ is an element in the base field GF(q). Thus

$$(\beta^{j_i e_{11}})^q = \beta^{j_i e_{11}} .$$

Hence we have $K_{j_i} = K_{j_i'}$. In other words, two different roots of $f_i(x)$, corresponding to b_{is_1} and b_{is_2}, will always give the same value of $K_j = q_o c_{11}\beta^{je_{11}}$, $0 \leq j \leq n-1$. By Definition 1 they will be in the same equivalent class.

In this special case, then, it is only necessary to calculate one j for each b_i, $1 \leq i \leq r$. This eliminates much of the calculations.

Case (2): $f(x) = (x-1)f_\nu(x)$, $1 \leq \nu \leq r$.

In this particular case we do not even have to apply Definition 1 at all. We have here all b_i, $i = 1,\ldots,r$ belonging to the same class. Thus if n is the number of appearance of a particular value b' which occurs most frequently in the set $\{b_1,b_2,\ldots,b_r\}$, and if we let $\zeta = r-n$, then the only condition we need to look for is whether $d_o < \zeta h$ is satisfied, after either condition (1) or (2) is met.

Case (3): $f(x) = (x-1)f_\nu(x)$, $1 \leq \nu \leq r$ and n a prime integer.

Under the circumstance of n being a prime integer x^n-1 will have factors all of the same degree except for the trivial factor of $x-1$. Hence we have $x^n-1 = (x-1)f_1(x)f_2(x)\ldots f_r(x)$, where each $f_i(x)$, $1 \leq i \leq r$, is irreducible and has degree h. In this case part of the constraints in conditions (1) and (2) are always met, namely the constraints $(n,q^i-1) = 1$ and $(n,q-1) = 1$.

The original result of Mattson-Solomon [1961] is a special case of this case.

Remarks

Theorem 3 shows a particular code may have a minimum distance one bigger than its BCH bound. In certain cases we may apply other results to our conclusion and obtain immediately a minimum distance at least as large as $d_o + 2$. We shall state some of these results.

First let us consider the special case of $q = 2$. In this case if condition (2) in Section II is met, then a minimum distance being equal to an even number implies $c_o = 0$. We can then factor out a power of x from $g_a(x)$ or $p_a(x)$ and check to see if any contradiction results. If the case turns out to be very special and meets Mattson-Solomon [1961] result, we can of course just apply their theorems.

Another result applicable to this end is a theorem given in a report by Mattson [1963]. The theorem is stated as follows.

Theorem 4:

(1) Let $p \equiv 1 \pmod 8$ be prime. Then for each n, $0 \leq 2n < p+1$, in the cyclic $(p, \frac{p+1}{2})$ code over GF(2), either both weights 2n-1 and 2n appear or neither appears.

(2) For $p \equiv -1$ and $3 \mid \frac{p+1}{4}$, then in the cyclic $(p, \frac{p+1}{2})$ code over GF(3), either both weights 3n-1 and 3n appear or neither appears (for $0 < 3n < p+1$).

Also Vera Pless in her paper [1963] mentioned the result of Prange and Gleasaon that may be also useful sometimes. Actually it is only applicable to the original Mattson-Solomon [1961] type of special case.

IV. EXAMPLES

In this section we shall present several different examples. The examples are so chosen that each will illustrate some particular points of the theory presented.

Example 1: Golay (11,6) Code over GF(3)

We all know that the linear Golay (11,6) code has minimum distance equal to 5. The BCH bound for its cyclic version, however, is only 4. We may apply our theory to show that the minimum distance of this cyclic code is 5. The details of this example has appeared elsewhere [Chien and Lum 1966].

Example 2: (73,10) Code over GF(2)

It was found that

$$x^{73} - 1 = (x-1)f_1(x)f_2(x)\dots f_8(x)$$

where $f_i(x)$ for $i = 1,2,\dots,8$ are irreducible over $GF(2)[x]$ and each one is of degree 9. We have here $h = 9$ and $m_o = 1$. The BCH bound for this code is $d_o = 25$. It was found that five coefficients of the 8^{th} degree terms in $f_i(x)$, $i = 1,2,\dots,8$ are 0's and three are 1's. Again we have special case (3) here. ζ as defined in Section III is thus equal to 3. Conditions of Theorem 3 are satisfied and $d_o = 25 < \zeta h = 3.9 = 27$. Thus the code has minimum distance $d \geq 25 + 1 = 26$. But if $d = 26$, then $c_o = 0$. Since for this code, $E(\beta) = \{0,25,50,27,54,35,70,67,61,49\}$, we have

$$g_a(x) = c_o + c_1x^{25} + c_2x^{50} + c_3x^{27} + c_4x^{54} + c_5x^{35} + c_6x^{70} + c_7x^{67} + c_8x^{61} + c_0x^{49} .$$

$c_o = 0$ implies

$$g_a(x) = x^{25}(c_1+c_2x^{25}+c_3x^2+\ldots+c_6x^{45}+\ldots+c_9x^{24}) .$$

The highest degree inside the parenthesis is 45, and therefore $g_a(x)$ can have at most 45 zeros. This means the minimum distance is at least 28, a contradiction to $d = 26$. Hence this code has minimum distance $d \geq 27$.

Example 4: (39,15) Code over GF(2)

The cycles for this code are

$$(1,2,4,8,16,32,25,11,22,5,10,20) \longleftrightarrow f_1(x)$$

$$(3,6,12,24,9,18,36,33,27,15,30,21) \longleftrightarrow f_2(x)$$

$$(7,14,28,17,34,29,19,38,37,35,31,23) \longleftrightarrow f_3(x)$$

$$(13,26) \longleftrightarrow \varphi_1(x)$$

$$(0) \longleftrightarrow (x-1)$$

Let $f(x) = (x-1)\varphi_1(x)\varphi_2(x)$, where $\varphi_2(x) = f_3(x)$, be the recursion polynomial for this code. Then we have $d_o = 7$, $m_o = 1$, $h_1 = 2$, and $h = 12$. Condition (2) is satisfied. We have according to our notation $e_{11} = 13$ and $e_{21} = 7$. We construct the following table for the application of Definition 1. (See Table 2.)

Applying Definition 1 we obtain three equivalent classes. These are

$$\rho_1 = \{b_3, b_3, b_1, b_3, b_3, b_1, b_3, b_3, b_1, b_1, b_1, b_1\}$$

$$\rho_2 = \{b_3, b_3, b_3, b_3, b_1, b_1, b_3, b_1, b_1, b_3, b_1, b_1\}$$

$$\rho_3 = \{b_2, b_2, b_2, b_2, b_2, b_2, b_2, b_2, b_2, b_2, b_2, b_2\}$$

It was found that $b_1 = 0$ and $b_3 = 1$. Thus we have $\sigma = 6 + 6 = 12$ and $d_o = 7 < \sigma = 12$. Hence by Theorem 3, the code has minimum distance $d \geq 8$.

There are numerous codes which the theory in Section III is applicable. Among the simple ones are the (23,12), (43,15), (113,29), (156,16) codes over GF(2), the (11,6), (19,10), (23,12) codes over GF(4) and different others over different finite fields.

Table 1

j	je_{11} mod 39	je_{21} mod 39	Corresponding b_i	j	je_{11} mod 39	je_{21} mod 39	Corresponding b_i
0	0	0	--	20	26	23	b_3
1	13	7	b_3	21	0	30	b_2
2	26	14	b_3	22	13	37	b_3
3	0	21	b_2	23	26	5	b_1
4	13	28	b_3	24	0	12	b_2
5	26	35	b_3	25	13	19	b_3
6	0	3	b_2	26	26	26	--
7	13	10	b_1	27	0	33	b_2
8	26	17	b_3	28	13	1	b_1
9	0	24	b_2	29	26	8	b_1
10	13	31	b_3	30	0	15	b_2
11	26	38	b_3	31	13	22	b_1
12	0	6	b_2	32	26	29	b_3
13	13	13	--	33	0	36	b_2
14	26	20	b_1	34	13	4	b_1
15	0	27	b_2	35	26	11	b_1
16	13	34	b_3	36	0	18	b_2
17	26	2	b_1	37	13	25	b_1
18	0	9	b_2	38	26	32	b_1
19	13	16	b_1				

Examples with $i_o \neq 1$

(i) $n = 113$, $k = 29$ over GF(2)

$h = 28$, $d_o = 17$, $m_o = 33$ (or 65)

(a) Take $m_o = 33$

$$m = 33 + 17 - 1 = 49$$

$$\begin{array}{l} \quad\;\; c_0\ c_1\ c_2\ c_3\ c_4\ c_5\ c_6\ c_7 \\ E(\beta) = \{0, 1, 2, 4, 8, 16, 32, 64, 15, 30, 60, 7, 14, 28, \\ \quad 56, 112, 111, 109, 105, 97, 81, 49, 98, 83, 53, \\ \quad 106, 99, 85, 57\} \end{array}$$

$$\begin{array}{l} E'(\beta) = \{64, 65, 66, 68, 72, 80, 96, 15, 79, 94, 11, 71, \\ \quad 78, 92, 7, 63, 62, 60, 56, 48, 32, 0, 49, 34, 4, \\ \quad 57, 50, 36, 8\}\ . \end{array}$$

Leading coefficient $= c_6 +$ h as degree $= 96$

Constant term $= c_6 q^{15} = c_{21}$

$i_o = 15$, $(i_o, h) = (15, 113) = 1$.

The other condition is satisfied since n = prime. The minimum distance of this code is ≥ 18.

(b) Take $m_o = 65$

$$m = 65 + 17 - 1 = 81$$

$$\begin{array}{l} E(\beta) = \{0, 1, 2, 4, 8, 16, 32, 64, 15, 30, 60, 7, 14, 28, \\ \quad 56, 112, 111, 109, 105, 97, 81, 49, 98, 83, 53, \\ \quad 106, 99, 85, 57\} \end{array}$$

$$E'(\beta) = \{32, 33, 34, 36, 40, 48, 64, 96, 47, 62, 92, 39, 46, 60, 88, 31, 30, 28, 24, 16, 0, 81, 17, 2, 85, 25, 18, 4, 89\} .$$

Leading coefficient $= c_7$

Constant term $= c_7{}^{q^{13}} = c_{20}$

$$i_o = 13 \quad , \quad (13,113) = (i_o,h) = 1.$$

So either choice of m_o will have $i_o \neq 1$.

(ii) $n = 19$, $k = 10$ over GF(4)

(1, 4, 16, 7, 9, 17, 11, 6, 5)

(2, 8, 13, 14, 18, 15, 3, 12, 10)

$h = 9$ $\quad d_o = 5$, $\quad m_o = 4$ (or 12).

(a) $m_o = 4$ $\qquad m = 4 + 5 - 1 = 8$

$$E(\beta) = \{0, 2, 8, 13, 14, 18, 15, 3, 12, 10\}$$

$$E'(\beta) = \{\underset{c_o}{11}, \underset{c_1}{13}, \underset{c_2}{0}, 5, 6, 10, 7, 14, 4, 2\} .$$

Leading coefficient $= c_7$

Constant term $= c_2 = c_7{}^{4^4}$

$$i_o = 4 \qquad (i_o,h) = (4,9) = 1.$$

(b) $m_o = 12$ $m = 12 + 5 - 1 = 16$

$$E(\beta) = \{0, 1, 4, 16, 7, 9, 17, 11, 6, 5\}$$

$$E'(\beta) = \{3, 4, 7, 0, 10, 11, 1, 14, 9, 8\} .$$

Leading coefficient $= c_8$

Constant term $= c_3$

$$C_8^{4^5} = c_3$$

$$i_o = 5, \qquad (i_o, h) = (5,9) = 1.$$

So either choice of m_o will give $i_o \neq 1$. The minimum distance of this code is ≥ 6.

Remarks

A number of additional BCH codes that satisfy the condition states in this paper and therefore possess a minimum distance greater than the BCH bound have been verified by Goethals [1966].

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In this paper the Mattson-Solomon algorithm is generalized in several directions. Based on the generalized algorithm, several classes of Bose-Chandhuri-Hocquenghem codes are given and shown to possess minimum distance of values greater than those given by the Bose-Chaudhuri-Hocquenghem found.

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